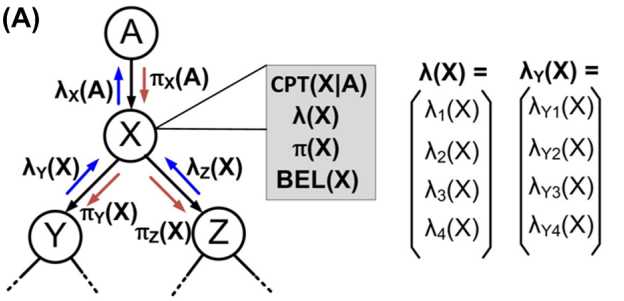
Architecting for Causal Intelligence at Nanoscale

Supplementary Document

(A)



$$\mathbf{BEL(X)} = [BEL(X=1) \quad BEL(X=2) \quad BEL(X=3) \quad BEL(X=4)]$$

$$\boldsymbol{\pi}\mathbf{(X)} = [\pi_1(X) \quad \pi_2(X) \quad \pi_3(X) \quad \pi_4(X)]$$

$$\boldsymbol{\pi}_X\mathbf{(A)} = [\pi_{X1}(A) \quad \pi_{X2}(A) \quad \pi_{X3}(A) \quad \pi_{X4}(A)]$$

$$\mathbf{CPT(X|A)} = \begin{pmatrix} P(X=1|A=1) & P(X=1|A=2) & P(X=1|A=3) & P(X=1|A=4) \\ P(X=2|A=1) & P(X=2|A=2) & P(X=2|A=3) & P(X=2|A=4) \\ P(X=3|A=1) & P(X=3|A=2) & P(X=3|A=3) & P(X=4|A=4) \\ P(X=4|A=1) & P(X=4|A=2) & P(X=4|A=3) & P(X=4|A=4) \end{pmatrix}$$

(B)

Bayesian Inference Operations using Pearl's Belief Propagation Algorithm	
Likelihood Estimation Prior Estimation	$\boldsymbol{\lambda}(\boldsymbol{X}) = \boldsymbol{\lambda_Y}(\boldsymbol{X}) * \boldsymbol{\lambda_Z}(\boldsymbol{X})$ $\boldsymbol{\pi}(\boldsymbol{X}) = \boldsymbol{\pi_X}(\boldsymbol{A}) \otimes \boldsymbol{CPT}(\boldsymbol{X}\|\boldsymbol{A})$
Belief Update	$\boldsymbol{BEL}(\boldsymbol{X}) = \alpha\boldsymbol{\pi}(\boldsymbol{X}) * \boldsymbol{\lambda}(\boldsymbol{X})$
<u>Support Messages:</u> Diagnostic Support to Parent Prior Support to Child Nodes	$\boldsymbol{\lambda_X}(\boldsymbol{A}) = \boldsymbol{CPT}(\boldsymbol{X}\|\boldsymbol{A}) \otimes \boldsymbol{\lambda}(\boldsymbol{X}),$ $\boldsymbol{\pi_Y}(\boldsymbol{X}) = \alpha\boldsymbol{\pi}(\boldsymbol{X}) * \boldsymbol{\lambda_Z}(\boldsymbol{X});$ $\boldsymbol{\pi_Z}(\boldsymbol{X}) = \alpha\boldsymbol{\pi}(\boldsymbol{X}) * \boldsymbol{\lambda_Y}(\boldsymbol{X}).$

Fig. S- 1. (A) Part of a BN with showing node X with parent A and child nodes Y, Z. All nodes have four states in this example. Each node maintains likelihood vector (**λ**), prior vector (**π**), belief vector (**BEL**), and conditional probability table (CPT). The CPT information and messages from child/parent nodes based on observed evidence are used to calculate **λ**, **π**, and **BEL** vectors during Bayesian inference. (B) Overview of computations involved in each node as per Belief propagation algorithm. Here elements in bold typeface indicate vectors as mentioned in (A). The symbol * indicates element-wise multiplication of vectors, which can be implemented using Multiplication Composer circuits. The symbol ⊗ indicates vector multiplication that has a sum-of-products form. These operations are implemented using Add-Multiply Composer circuits.

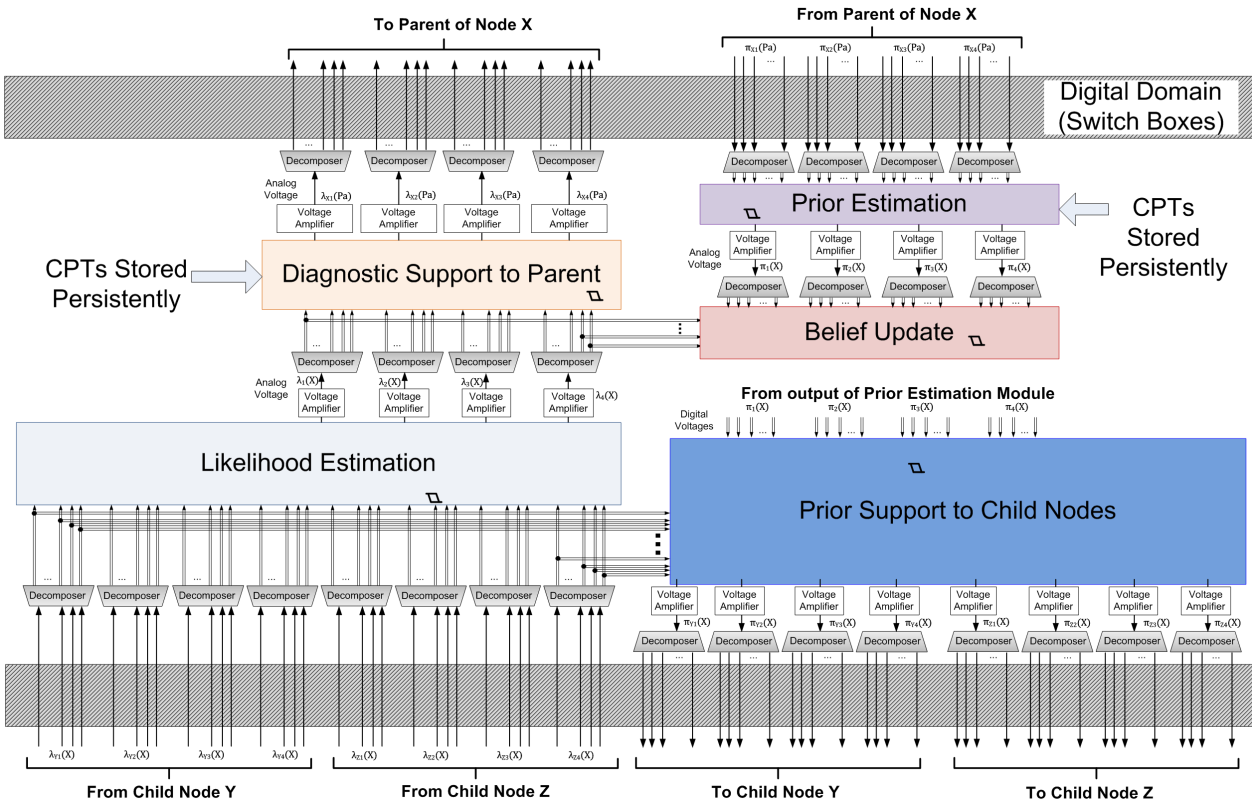


Fig. S- 2. Detailed schematic of the Bayesian Cell showing various modules involved in inference operation. They use Multiplication Composers and Add-Multiply Composers, cascaded through Decomposers [8]. The likelihood, prior and belief vectors, and conditional probability tables (CPTs) are stored persistently in each module.

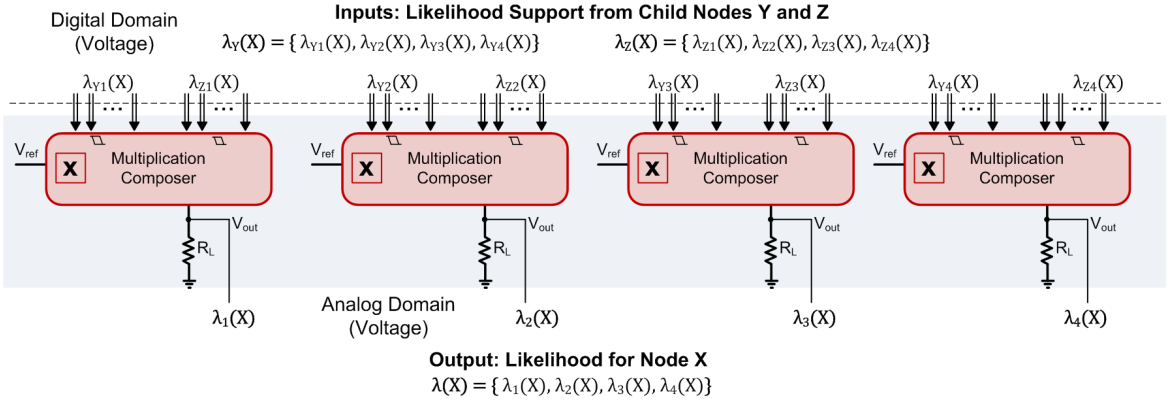


Fig. S- 3. Implementation of Likelihood Estimation operation using Multiplication Composers for a node X, supporting up to four states. Inputs are likelihood support message vectors $\boldsymbol{\lambda_Y(X)}$ and $\boldsymbol{\lambda_Z(X)}$ from child nodes Y and Z respectively. Each vector has four elements corresponding to each state of node X. Belief Update and Prior Support modules use similar implementation.

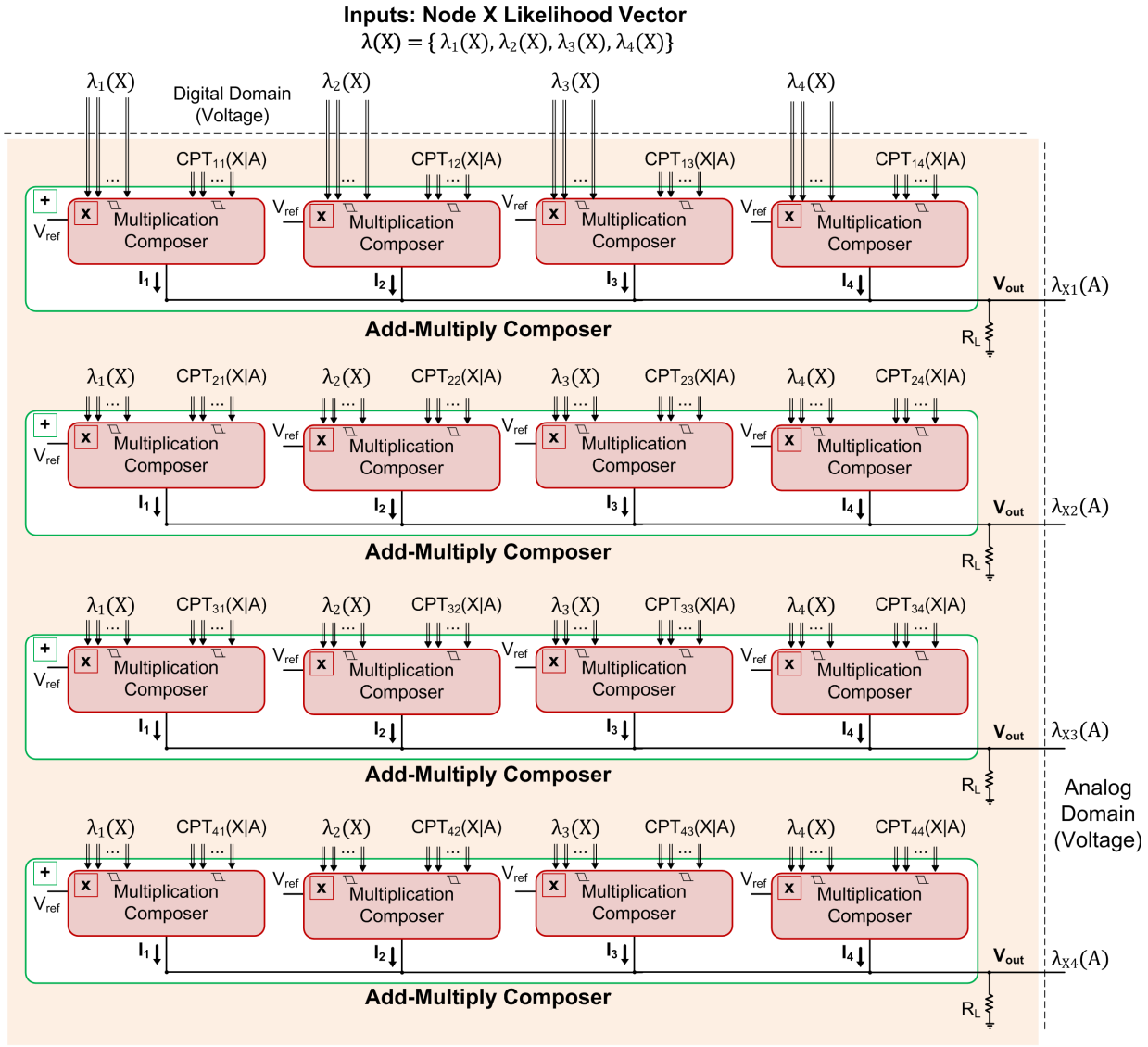


Fig. S- 4. Implementation of Diagnostic Support module using Add-Multiply Composers for a node X. The input is a likelihood vector **λ(X)** for node X. The output is a diagnostic support message vector $\boldsymbol{\lambda_X(A)}$ to be propagated to parent node A. Each vector has four elements corresponding to each state of node X and A. The conditional probability table (CPT) elements are stored persistently in the Composers, obviating the need to interface with external memory. Prior Estimation module similarly uses Add-Multiply Composers.

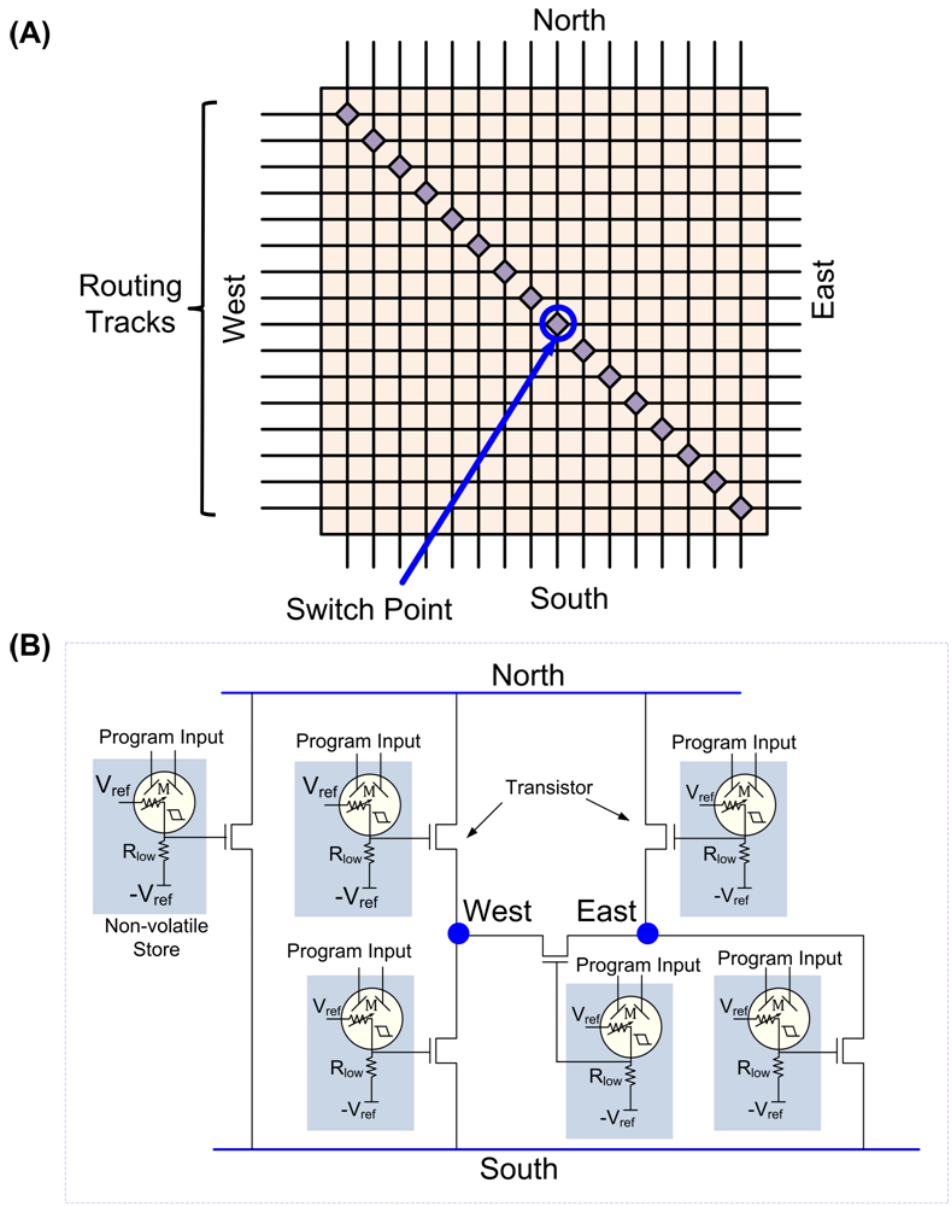


Fig. S- 5. (A) Switch Box schematic: It is a collection of *switch-points*, where each switch-point is capable of routing an incoming signal from one direction to any of three outgoing directions. (B) Switch-point schematic for programmable routing. It consists of six non-volatile S-MTJs gating pass transistors (Metal Oxide Semiconductor Field Effect Transistors - MOSFETs), to be able to activate/deactivate connections between the four directions shown. S-MTJ storing a high resistance deactivates a link, while a low resistance state activates the link allowing signal propagation.